Constraining the fundamental interactions and couplings with Eötvös experiments

Lucila Kraiselburd[1][*]and Hector Vucetich [1][†]

[1]*Grupo de Gravitación, Astrofísica y Cosmología,*
Facultad de Ciencias Astronómicas y Geofísicas,
Universidad Nacional de La Plata,
Paseo del Bosque S/N (1900) La Plata, Argentina

May 23, 2012

Abstract

Upper bounds for the violation of the Weak Equivalence Principle (WEP) by the Fundamental Interactions have been given before. We now recompute the limits on the parameters measuring the strength of the violation with the whole set of high accuracy Eötvös experiments. Besides, limits on spatial variation of the Fundamental Constants α, $\sin^2 \theta_W$ and v, the vacuum expectation value of the Higgs field, are found in a model independent way. Limits on other parameters in the gauge sector are also found from the strucutre of the Standard Model.

1 Introduction

The Standard Model of Fundamental Interactions (SM) together with General Relativity (GR) provide a consistent description of all known local low energy phenomena (i.e. low compared to the Grand Unified (GU) energy scale) in good agreement with the experiment. These theories depend on a set of parameters called the "fundamental constants", which are supposed to be universal parameters; i.e.: time, position and reference frame invariant [1, 2].

The Equivalence Principle (EP) is the physical basis of gravitational theory. Briefly, it states that a freely falling reference frame is equivalent to an inertial reference frame [3]. The unrestricted validity of this very strong statement implies that General Relativity is the unique theory of the gravitational field [4]. Thus, experimental tests of its validity probe deeply into the structure of gravitation.

The traditional model for describing an Equivalence Principle breakdown is to assume that an anomalous acceleration of body A, defined as $\delta \boldsymbol{a}_A = \boldsymbol{a}_A - \boldsymbol{g}$ is due to a difference between its inertial mass m_A^I and its passive gravitational mass m_A^P, i.e. the coupling constant of A to the gravitational field. It is usual

[*]lkrai@fcaglp.fcaglp.unlp.edu.ar
[†]vucetich@fcaglp.unlp.edu.ar

1

to parametrize δm_A in the form [5]

$$\delta m_A = m_A^P - m_A^I = \sum_K \Gamma_K \frac{E_K}{c^2}, \qquad (1)$$

where E_K are the different contributions to the binding energy of A, and Γ_K are dimensionless parameters quantifying the breakdown of UFF.

One of the consequences of the Equivalence Principle is that the fundamental constants must be universal parameters, because any dependence on time, position or reference frame would break the equivalence with an inertial frame [6, 7]. The particular case of space dependence of dimensionless constants has been treated in those references and with weaker hypotheses (essentially energy conservation) in reference [8].

In short, the binding energies E_K of bodies such as nuclei are functions of the fundamental constants, and each gives a contribution $\frac{E_K}{c^2}$ to the mass. If the fundamental constants are space dependent, so is the mass of the body. In those conditions, the Lagrangian of a body in a gravitational field takes the form

$$L = - \int m(\alpha) \sqrt{g_{\mu\nu} u^\mu u^\nu} ds. \qquad (2)$$

In the nonrelativistic limit one finds an anomalous acceleration

$$\delta \boldsymbol{a} = - \sum_j \frac{c^2}{m} \frac{\partial m}{\partial \alpha_j} \boldsymbol{\nabla} \alpha_j, \qquad (3)$$

where j runs over the set of fundamental constants. This anomalous acceleration is composition-dependent and its existence can be tested through Eötvös experiments.

Recently, the detection of a spatial variation of the fine structure constant has been reported [9, 10, 11] with a gradient amplitude $(3.6 \pm 0.6) \times 10^{-6}\,\mathrm{Gpc}^{-1}$ at a $\sim 6\sigma$ level. This tantalizing result suggests that local variation of α should be tested via local experiments. And the Eötvös experiment (Section 2) offers an excellent tool for that. Indeed, Dent [12] has made such an analysis (See also, [2, 13]).

The purpose of this paper is to analyze the space variation of the fundamental constants in the SM, using the available Eötvös experiment results (Cf. Table 1). We shall limit ourselves to the Gauge sector of the Standard Model, with the exception of the vacuum expectation value of the Higgs field. The organization is as follows: Section 2 summarizes the main characteristics and results of the Eötvös experiments, and describes the models we shall use for our analysis. Section 3 delineates our implementation of the structural characteristics of the test bodies such as binding energies and constitutive relations in the Standard Model. Section 4 shows our results and in Section 5 we state our conclusions.

2   A primer on Eötvös experiments

The Eötvös experiment [14], one of the most sensitive tests of the Equivalence Principle, measures the difference of acceleration between two masses A, B in the same gravitational field. It consists in suspending a pair of bodies from the

Body	Body	Source	$(\eta \pm \sigma) \times 10^{11}$	Ref.
Al	Au	Sun	1.3±1.5	[15]
Al	Pt	Sun	0.03±0.04	[16]
Cu	W	Sun	0.6±2.0	[17]
Be	Al	Earth	−0.02±0.28	[18]
Be	Cu	Earth	−0.19±0.25	[18]
Si/Al	Cu	Sun	0.51±0.67	[18]
Moon-Like	Earth-Like	Sun	0.005±0.089	[19]
Be	Ti	Earth	0.003±0.018	[20]
Be	Al	Earth	−0.015±0.015	[21]
Be	Ti	Sun	−0.031±0.045	[21]
Be	Al	Sun	0.0±0.042	[21]

Table 1: Results of the Eötvös experiment

arms of a torsion balance in a homogeneous gravitational field. It is easy to show that a differential acceleration would produce a torque [4, 5]

$$T = LW\eta(A, B), \tag{4}$$

being L and W the lever arm of the torsion balance and the gravitational force on the body respectively, and $\eta(A, B)$ the Eötvös parameter. The torsion balance is rotated with a well defined angular velocity ω with respect to the external gravitational field and only signals with the corresponding period are analyzed in order to clean the result of spurious systematic effects. Additionally, it would be possible to find any "privileged direction" defined by a gradient in the masses if a nonzero result were found.

The main result of the Eötvös experiment is the Eötvös parameter $\eta(A, B)$, defined as follows: If $\boldsymbol{g}$ is the local acceleration of gravity,

$$\eta(A, B) = \frac{(\boldsymbol{a}_A - \boldsymbol{a}_B) \cdot \boldsymbol{n}}{|\boldsymbol{g}|}, \tag{5}$$

where $\boldsymbol{a}_{A,B}$ are the accelerations of the bodies in the gravitational field and $\boldsymbol{n}$ a suitably chosen unit vector. Since the Equivalence Principle implies that $\boldsymbol{a}_A = \boldsymbol{a}_B = \boldsymbol{g}$, a non-null η signals its breakdown. The beautiful design of the experiment cancels many causes of error and during the twentieth century several orders of magnitude in accuracy have been improved. Table 1 displays the results of several high accuracy Eötvös experiments.

Equation (4) depends crucially on the homogeneity of the gravitational field $\boldsymbol{g}$ and great efforts have been made to design the torsion balance so that its small inhomogeneities are canceled. Besides, due to the design, the Eötvös experiment is sensitive only to the horizontal component of the gravitational field. Thus, only the deviation of the Earth gravitational field $g_\perp$ or the solar gravitational field $g_\odot$ are used in the experiments. The references cited in Table 1 include many details on the design of the experiment and the analysis of experimental data.

With model (1), the Eötvös parameter reads

$$\eta(A, B) = \frac{\delta m_A}{m_A} - \frac{\delta m_A}{m_A} = \sum_K \Gamma_K \left(\frac{E_K^{(A)}}{Mc^2} - \frac{E_K^{(B)}}{Mc^2} \right), \tag{6}$$

3

and a set of experiments with different compositions permit in principle the measurement of the Γ_K parameters.

Let

$$M(A, Z) = m_p Z + m_n N + m_e Z - \frac{B(Z, A)}{c^2} \qquad (7)$$

be the atomic mass of a body of mass number A, atomic number Z, neutron number $N = (A - Z)$ and binding energy $B(Z, A)$. The difference between inertial and passive gravitational mass of the above body will be [22]

$$\begin{aligned}
\delta M &= \delta m_p Z + \delta m_n N + \delta m_e Z - \delta \frac{B(Z, A)}{c^2} \\
&= \frac{\delta(m_p + m_n + m_e)}{2} A + \delta Q \frac{(N - Z)}{2} - \delta \frac{B(Z, A)}{c^2},
\end{aligned} \qquad (8)$$

where

$$Q = m_n - m_p - m_e \qquad (9)$$

is the decay energy of the neutron. The relative mass difference will be

$$\frac{\delta M}{M} \simeq \frac{\delta(m_p + m_n + m_e)}{2m_p} + \frac{N - Z}{2A} \frac{\delta Q}{m_p} - \delta \frac{B(Z, A)}{A m_p c^2}. \qquad (10)$$

Finally, we shall parametrize $\delta \frac{B(Z,A)}{A m_p c^2}$ either in the "classical" form (1) for a test of the Equivalence Principle, or in the form (3) for testing the position dependence of the fundamental constants. In the last case, the Eötvös parameter will read, after some algebra,

$$\eta(A, B) = \frac{c^2}{g} \sum_j \frac{\partial \ln \frac{M^{(A)}}{M^{(B)}}}{\partial \ln \frac{\alpha_j}{\Lambda_j}} \boldsymbol{n} \cdot \boldsymbol{\nabla} \ln \frac{\alpha_j}{\Lambda_j} \qquad (11)$$

being Λ_j suitable normalization constants.

3 Binding energies and fundamental constants

The main ingredients for our analysis are the binding energies E_K and their dependence on the fundamental constants. We shall discuss separately nuclear binding energies and neutron-proton mass differences.

3.1 Nuclear binding energies

The largest contribution to the binding energy of an atom comes from the nuclear binding, which has been discussed for a long time. The simplest approach is to use the semi-empirical mass formula [23, 24] complemented with the estimate of the weak interactions contribution to the binding energy [25, 26]. There are simple analytic approximations for the strong, Coulomb and weak contributions

to the binding energy B, namely

$$\frac{E_S}{Mc^2} = a_V - a_S A^{-1/3} - a_A \frac{(N-Z)^2}{A^2}, \tag{12a}$$

$$\frac{E_C}{Mc^2} = \frac{3e^2}{5r_0} \frac{Z(Z-1)}{A^{4/3}}, \tag{12b}$$

$$\frac{E_W}{Mc^2} = \mathcal{G}G_F 2^{-2/3} V^{-1} \left\{ NZ \left[(3\alpha_\beta^2 - 1) - 4 \left(\frac{1}{2} - 2 * \sin^2\theta_W \right) \right] \right. \\ \left. + \frac{N^2}{2} + \frac{Z^2}{2} (4\sin^4\theta_W - 2\sin^2\theta_W + 1) \right\}. \tag{12c}$$

In the above equations $r_0 A^{1/3}$ is the nuclear radius, $V = {}^{4\pi}/_3 A r_0^3$ the nuclear volume and $N = A - Z$ the neutron number. α_β is the ${}^{G_A}/_{G_V}$ ratio for neutron decay, and G_F and θ_W are the Fermi constant and the Weinberg angle respectively. $\mathcal{G}$ is an enhancement factor of the weak interactions due to the strong ones [26]. Besides, the "strong constants" a_V, a_S, a_A as well as m_p, m_n are all proportional to Λ_{QCD} in the chiral limit.

These analytic expressions, which are reasonably accurate, display the dependence of the nuclear binding energies on the fundamental constants.

3.2 Neutron-proton mass difference

The other contribution to the mass is the neutron-proton mass difference which contributes to the neutron decay energy Q.

Model independent contribution of the strong, electromagnetic and weak forces to the neutron-proton mass difference ΔM can be computed with the Cottingham formula [27] and its generalizations for the strong [28] and weak [22] interactions. Their calculated values are:

$$\left. \frac{\Delta M}{M} \right|_S = 2.22 \times 10^{-3}, \quad \left. \frac{\Delta M}{M} \right|_E = -0.83 \times 10^{-3}, \quad \left. \frac{\Delta M}{M} \right|_W = -5.0 \times 10^{-9}. \tag{13}$$

However, the explicit dependence on the fundamental constants is not obvious. A careful analysis of the respective expressions shows that the electromagnetic contribution is proportional to α and the weak one to G_F. Besides, the weak contribution has a dependence on $\sin^2\theta_W$, which must be numerically computed with reference [22] method. The result is

$$\frac{\sin^2\theta_W}{M} \frac{\partial \Delta M}{\partial \sin^2\theta_W} \simeq 2.0 \times 10^{-8}. \tag{14}$$

Finally, an important result is that the "strong" contribution to ΔM is not proportional to Λ_{QCD} near the chiral limit but to the $u-d$ quarks mass difference, a result that can be derived in an elementary way from Chiral Perturbation Theory [29] and that is quantitatively confirmed in lattice calculations (See, for instance, [30]).

Since quark and electron masses are proportional to the the vacuum expectation value of the Higgs field v, $m_i = y_i v$, so is Q. The available Eötvös

experiments are not enough to separate the Yukawa coupling parameters and v. So in this paper we limit ourselves to the analysis of the gauge sector plus the single Higgs sector parameter v. With this limitation, we find the following expression for Q as a function of the fundamental constants α, v and $\sin^2\theta_W$:

$$\frac{\delta Q}{M} = \frac{\delta\alpha}{\alpha}\left.\frac{\Delta M}{M}\right|_E + \frac{\delta v}{v}\frac{Q}{M} + \frac{\sin^2\theta_W}{M}\frac{\partial\Delta M}{\partial\sin^2\theta_W}\frac{\delta\sin^2\theta_W}{\sin^2\theta_W} \qquad (15)$$

If for each of the fundamental constants α_i we replace

$$\delta\alpha_i = \boldsymbol{\nabla}\boldsymbol{\alpha}_i \cdot \delta\boldsymbol{r} \qquad (16)$$

we obtain the contribution of Q to the Eötvös parameter.

3.3 Constitutive relations

In this subsection we shall use the fine structure constant α, the vacuum expectation value of the Higgs field v and the squared sine of Weinberg's angle θ_W as our basic variables. Other fundamental constants from the gauge sector are related to our basic constants in the form

$$\alpha = \alpha_1\sin^2\theta_W, \qquad \tan^2\theta_W = \frac{\alpha_2}{\alpha_1}, \qquad G_F = \frac{1}{\sqrt{8}v^2}, \qquad (17a)$$

$$M_W^2 = \frac{\alpha_1}{2}v^2, \qquad M_Z^2 = \frac{M_W^2}{\cos^2\theta_W}, \qquad \alpha_3 = \frac{\beta^{-1}}{\ln\frac{\mu^2}{\Lambda_{\mathrm{QCD}}^2}}. \qquad (17b)$$

The last equation shows that QCD system of units, α_3 is automatically constant.

3.4 Scaling and systems of units

The need of working with nondimensional quantities when studying the variation of fundamental constants it has been discussed in many papers (See, for instance, [1, 2]) since a suitable choice of units may cancel its variation. Many measurements, however, are carried out on dimensional quantities and its analysis must be done starting with these data. This problem may be solved either by transforming the dimensional quantities to a standard system of units [31] or transforming these dimensional quantities into dimensionless ones through division by a suitably chosen constant.

One of the beauties of the Eötvös experiment is that it has a "natural" way of defininig η as nondimensional parameter, and equation (6) is already in dimensionless form. Besides, since the anomalous acceleration (3) can be written as

$$\delta\boldsymbol{a} = -\sum_j \frac{c^2}{m}\frac{\partial m}{\partial\alpha_j}\boldsymbol{\nabla}\ln\alpha_j. \qquad (18)$$

Any normalization constant will not contribute to the differential acceleration.

In this paper we shall use "QCD units": that is, we assume that

$$\boldsymbol{\nabla}\Lambda_{\mathrm{QCD}} = 0. \qquad (19)$$

This implies that $\Gamma_S = 0$ since we are working near the chiral limit.

Param	$\Gamma \pm \sigma(\Gamma)$	Correlations			Up. Bound
Γ_S	$(-1.3\pm1.6) \times 10^{-10}$	1.00	0.98	0.84	5×10^{-10}
Γ_C	$(-3.0\pm4.0) \times 10^{-10}$	0.98	1.00	0.87	1.2×10^{-9}
Γ_W	$(-2.7\pm8.6) \times 10^{-4}$	0.85	0.87	1.00	2.6×10^{-3}
Γ_C	$(1.5\pm6.7) \times 10^{-11}$		1.00	0.00	2×10^{-10}
Γ_W	$(2.2\pm2.0) \times 10^{-4}$		0.00	1.00	6×10^{-4}

Table 2: Test of the Equivalence Principle.

Param	$\Gamma \pm \sigma(\Gamma)$	Correlations			Up. Bound
Θ_α	$(0.5\pm1.6) \times 10^{-10}$	1.00	0.74	0.71	4.8×10^{-10}
Θ_v	$(-0.5\pm2.5) \times 10^{-8}$	0.74	1.00	0.99	1.1×10^{-7}
$\Theta_{\sin^2 \theta_W}$	$(0.1\pm1.1) \times 10^{-2}$	0.71	0.99	1.00	3.3×10^{-2}

Table 3: Bounds on the space variation of fundamental constants: Basic variables.

4 Results

We have performed the above sketched calculations both to test the Equivalence Principle and the existence of gradients of the fundamental constants in the Standard Model. A weighted least squares procedure was applied to the values of η in Table 1, the conditional equation being given by either equation (6) or in the form corresponding to spatial variation (11).

4.1 Test of the Equivalence Principle

Table 2 shows our results for the test of the Equivalence Principle. We have used the expressions (12) with the contributions (13) from the neutron-proton mass difference. The first three lines of the table show the result assuming that the three interactions break the Equivalence Principle. The last two lines assume that the strong contribution satisfies the Equivalence Principle ($\Gamma_S = 0$). An enhancement factor $\mathcal{G} = 8$ for the Weak Interactions has been assumed [22]. Our results are similar to those of reference [22], but the new bounds are smaller due to the inclusion of higher accuracy results [19, 20, 21].

The large correlations in the first three lines suggest that either the breakdown of the Equivalence Principle should be analyzed simultaneously or that a constraint such as ($\Gamma_S = 0$) should be imposed and in this case only violation parameters relative to the strong interactions will be found.

4.2 Spatial variation of Fundamental constants

Turning to the spatial variation problem, it is convenient to work with the nondimensional quantity [13]

$$\Theta_j = \frac{c^2}{g_\odot} \frac{|\boldsymbol{\nabla}\alpha_j|}{\alpha_j}, \tag{20}$$

which is the "natural" nondimensional parameter for this problem. As explained before, we use as basic variables α, v and $\sin^2 \theta_W$. Again, the values of the Θ parameters were found by least squares adjustment and upper bounds were

j	Θ_j	$\ell_j$pc	j	Θ_j	$\ell_j$pc
α	4.8×10^{-10}	1.1×10^{12}	v	7.5×10^{-8}	6.6×10^{9}
$\sin^2 \theta_W$	3.4×10^{-2}	1.4×10^{4}	α_1	3.4×10^{-2}	1.4×10^{4}
α_2	1.0×10^{-2}	4.8×10^{4}	G_F	1.5×10^{-7}	3.4×10^{9}
M_W^2	3.4×10^{-2}	1.4×10^{4}	M_Z^2	2.4×10^{-2}	2.0×10^{4}

Table 4: Bounds on the space variation of fundamental constants: Gauge sector.

obtained as 3σ values. The results of the adjustment are displayed in Table 3 in the same format as the one in Table 2.

Logarithmic differentiation of the Standard Model relations in (17), after normalization by division by suitable powers of Λ_{QCD}, yields a system of linear equations for the gradients of the parameters from which the upper bounds of Table 4 are found. The quantities

$$\ell_j = \frac{\alpha_j}{|\boldsymbol{\nabla}\alpha_j|} \tag{21}$$

define distance scales where the spatial variation of a given fundamental constant becomes important.

The results summarized in Tables 2 to 4 are the main results of this paper.

5   Conclusion

The results stated in Sections 2 and 4 show that no violation of the Equivalence Principle is observable in laboratory experiments down to the 10^{-13} level. The classical model decomposition of the Eötvös parameter (6) shows that the contributions of the fundamental interactions to such a violation are extremely small.

On the other hand, the order of magnitude of the upper bounds for the gradients of the fundamental constants are very variable, from $\sim 10^{-12}\,\mathrm{pc}^{-1}$ for α to $\sim 10^{-4}\,\mathrm{pc}^{-1}$ for α_2. These extremely small gradients of galactic or cosmological scale, are the best available bounds on the spatial variation of fundamental constants.

The results of this paper are in a certain sense complementary to those of reference [12] where the analysis was focused mainly on the Higgs sector of the Standard Model and on the sensitivity to the Newtonian potential. See also reference [2] for a more complete analysis of that sector.

Our upper bounds, however, are too big for an independent test of the reported cosmological gradient of α. Our smallest bound is obtained assuming that only α has a sensible variation

$$\frac{|\boldsymbol{\nabla}\alpha_j|}{\alpha_j} < 2 \times 10^{-4}\,\mathrm{Gpc}^{-1}, \tag{22}$$

and it is about 60 times greater than the detected one. This is not far from the needed sensitivity and the proposed MICROSCOPE [32] or STEP [33] experiments, whose accuracy is about a thousand times greater; should be able to detect it.

Acknowledgments

References

[1] Jean-Philippe Uzan. The Fundamental constants and their variation: Observational status and theoretical motivations. *Rev.Mod.Phys.*, 75:403, 2003.

[2] Jean-Philippe Uzan. Varying Constants, Gravitation and Cosmology. *Living Rev.Rel.*, 14:2, 2011. 145 pages, 10 figures, Review for Living Reviews in Relativity.

[3] A. Einstein. Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes. *Ann. Phys. (German)*, 35:898, 1911.

[4] S. Weinberg. *Gravitation and Cosmology*. Wiley, 1972.

[5] C. M. Will. *Theory and Experiment in Gravitational Physics*. Cambridge University Press, March 1993.

[6] P. J. Peebles and R. H. Dicke. Significance of Spatial Isotropy. *Physical Review*, 127:629–631, July 1962.

[7] R. H. Dicke. Experimental relativity. In C. deWitt and B. deWitt, editors, *Relativity, Groups and Topology*. Grodin and Breach, 1964.

[8] M. P. Haugan. Energy conservation and the principle of equivalence. *Annals of Physics*, 118:156–186, March 1979.

[9] J. K. Webb, J. A. King, M. T. Murphy, V. V. Flambaum, R. F. Carswell, and M. B. Bainbridge. Indications of a Spatial Variation of the Fine Structure Constant. *Physical Review Letters*, 107(19):191101, November 2011.

[10] J. A. King, J. K. Webb, M. T. Murphy, V. V. Flambaum, R. F. Carswell, M. B. Bainbridge, M. R. Wilczynska, and F. Elliot Koch. Spatial variation in the fine-structure constant – new results from VLT/UVES. *ArXiv e-prints*, February 2012.

[11] J. C. Berengut, E. M. Kava, and V. V. Flambaum. Is there a spatial gradient in values of the fine-structure constant? A reanalysis of the results. *ArXiv e-prints*, March 2012.

[12] Thomas Dent. Eotvos bounds on couplings of fundamental parameters to gravity. *Phys.Rev.Lett.*, 101:041102, 2008.

[13] H. Vucetich. Time variation of fundamental constants: two phenomenological models. In U. Cotti, M. Mondragón, & G. Tavares-Velasco, editor, *Particles and Fields*, volume 670 of *American Institute of Physics Conference Series*, pages 298–305, June 2003.

[14] R. V. Eötvös, D. Pekár, and E. Fekete. Beiträge zum Gesetze der Proportionalität von Trägheit und Gravität. *Annalen der Physik*, 373:11–66, 1922.

[15] P. G. Roll, R. Krotkov, and R. H. Dicke. The equivalence of inertial and passive gravitational mass. *Ann. Phys. (N. Y.)*, 26:442, 1964.

[16] V. B. Braginski and V. I. Panov. Verification of the equivalence of inertial and gravitational mass. *Sov. Phys. J. E. T. P.*, 34:463, 1972.

[17] G. M. Keiser and J. E. Faller. A new approach to the Eötvös experiment. In R. Ruffini, editor, *Proc. 2nd Marcel Grossman Meeting on General Relativity*, page 969, 1982.

[18] Y. Su, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, M. Harris, G. L. Smith, and H. E. Swanson. New tests of the universality of free fall. *Physical Review D*, 50:3614–3636, September 1994.

[19] S. Baeßler, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt, and H. E. Swanson. Improved Test of the Equivalence Principle for Gravitational Self-Energy. *Physical Review Letters*, 83:3585–3588, November 1999.

[20] S. Schlamminger, K.-Y. Choi, T. A. Wagner, J. H. Gundlach, and E. G. Adelberger. Test of the Equivalence Principle Using a Rotating Torsion Balance. *Physical Review Letters*, 100(4):041101, February 2008.

[21] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl, and S. Schlamminger. Torsion balance experiments: A low-energy frontier of particle physics. *Prog. Part. Nucl. Phys.*, 62:102–134, 2009.

[22] N. Chamoun and H. Vucetich. Weak interactions effect on the P - N mass splitting and the principle of equivalence. *Phys.Lett.*, B541:291–297, 2002.

[23] C. F. von Weiszäcker. Zur theorie der kernmassen. *Zeitschrift für Physik*, 96:35, 1935.

[24] B. Martin. *Nuclear and Particle Physics, An Introduction*. Wiley, New York, 2nd edition, 2009.

[25] Mark P. Haugan and Clifford M. Will. Weak interactions and the Eötvös experiment. *Phys. Rev. Lett.*, 37:1, 1976.

[26] Ephraim Fishbach, Mark P. Haugan, Dubravko Tadić, and Hai-Yang Cheng. Lorenz noninvariance and the Eötvös experiments. *Phys. Rev. D*, 32:154, 1985.

[27] W. N. Cottingham. The neutron proton mass difference and electron scattering experiments. *Annals Phys.*, 25:424–432, 1963.

[28] L. N. Epele, H. Fanchiotti, C. A. García Canal, and R. Méndez Galain. Phenomenological analysis of the neutron-proton mass difference. In H. M. Nussenzveig S. MacDowell and R. A. Salmeron, editors, *Frontier Phisics: Essays in Honour of Jayme Tiomno*, page 57, 1991.

[29] J. Gasser and H. Leutwyler. Quark masses. *Physics Reports*, 87:77–169, July 1982.

[30] Silas R. Beane, Kostas Orginos, and Martin J. Savage. Strong-isospin violation in the neutron proton mass difference from fully-dynamical lattice QCD and PQQCD. *Nucl. Phys.*, B768:38–50, 2007.

[31] P. Sisterna and H. Vucetich. Time variation of fundamental constants: Bounds from geophysical and astronomical data. *Physical Review D*, 41:1034–1046, February 1990.

[32] Microscope: Microsatellite for the test of the equivalence principle. `http://smsc.cnes.fr/MICROSCOPE/index.htm`.

[33] Step: Satellite test of the equivalence principle. `http://einstein.stanford.edu/STEP/`.